Master theses in Numerical Analysis

Subjects

1 Parallel aggregation with quality control.

[Yvan Notay (ynotay@ulb.ac.be), Artem Napov (anapov@ulb.ac.be)]

2 Coarse grid solver for parallel multigrid methods: large aggregates vs. domain decomposition.

[Yvan Notay (ynotay@ulb.ac.be), Artem Napov (anapov@ulb.ac.be)]

3 Algebraic multigrid methods for electromangetism: novel aggregation strategies.

[Artem Napov (anapov@ulb.ac.be), Yvan Notay (ynotay@ulb.ac.be)]

Context

The master thesis subjects listed above are related to **multigrid methods**. These methods are used for the solution of large linear systems of equations, with typically several millions to several billions of unknowns. The *Service de Métrologie Nucléaire* team has an internationally recognized expertise in the field of multigrid methods. In particular, the team develops the AGMG code, which has hundreds of users around the world.

The basic principle behind multigrid methods is to solve a given linear system using a hierarchy of decreasing in size linear systems. The above subjects within this framework are in general related to *aggregation-based* multigrid methods, which construct a hierarchy of decreasing in size linear systems by grouping unknowns into aggregates ; this yields simple yet efficient multigrid variants. Because the aggregates are automatically formed on the basis of the linear system matrix, these methods are on the other hand much more flexible than the multigrid methods based on a predefined hierarchy of grids.

All master thesis subjects are directly connected to the research activities of the team.

Short Description : next pages

1 Parallel aggregation with quality control

The AGMG software implements a multigrid method based on the automatic aggregation of the unknowns, with the peculiarity that the "quality" of the aggregates is checked and partly optimized during the aggregation phase ; "quality" here refers to a mathematical quantity which is related to the global convergence of the method.

As a drawback, the aggregation has to proceed in a purely sequential greedy fashion. This prevents from porting the aggregation phase to massively parallel computational units like GPUs. For such devices, there exit fast implementations of aggregation processes known under the name of "matching algorithms". However, these are not specifically designed for multigrid methods, and may result in aggregates of poor quality.

The aim of the proposed thesis is to develop an aggregation technique that combines the fast matching algorithms with quality control.

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2 Coarse grid solver for parallel multigrid methods: large aggregates vs. domain decomposition

In its parallel version, the AGMG software solves the system on the coarsest grid using a two-level iterative method based on the inversion of the "local parts" of the matrix in combination with a very aggressive aggregation (a super coarse grid is formed by grouping into one aggregate all unknowns associated to the same processor). This method is in fact somewhat similar to a two-level domain decomposition method, in which one also uses exact inversion of "subdomain matrices". However, in this latter approach, the subdomains overlap, while the matrix blocks corresponding to "local parts" are disjoint.

The work consists in exploring the similarities and differences between both approaches, in particular in analyzing how the convergence is affected when one adds overlapping to the existing method in AGMG.

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3 Algebraic multigrid methods for electromangetism: novel aggregation strategies

The focus of the proposed work is on the solution of linear systems of equations arising from the disretization of boundary value problems of curl-curl type, that is, problems in which the highest order term corresponds to a double application of the **curl** differential operator. The solution of such systems is a key element of numerous simulations in the field of electromagnetism. These systems are characterized by the kernel of the discrete **curl** operator, that corresponds (in the case of simply connected domain) to the space of discrete gradients.

The considered solution method is of aggregation-based algebraic multigrid type. These iterative methods build a hierarchy of grids by grouping unknowns into aggregates, and subsequently use this hierarchy to ensure a rapid convergence. For the curl-curl problems, the aggregation is first performed for the auxiliary unknowns associated with discrete gradients, and the actual aggregates are then deduced from this latter.

The aggregation in the space of discrete gradients is typically based on the projection of the system matrix into this space, projection whose result is, paradoxically, independent on the curl-curl term (since for a regular enough $\mathbf{u}$ one has $\mathbf{curl}\,\mathbf{grad}\,\mathbf{u} = \mathbf{0}$). This approach is therefore problematic if the low-order terms are either missing (since the result of the projection is then nil) or are not representative of the curl-curl term. Based on the recent convergence analysis of the considered method, it is proposed to develop an alternative approach which is based on the non-projected system matrix. The work includes the use of theoretic tools, algorithmic developments, implementation and evaluation of the proposed approaches.

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