# Master theses in Numerical Analysis

## Subjects

### 1 Algebraic multigrid methods for electromangetism: novel aggregation strategies.

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## Context

The master thesis subjects listed above are related to **multigrid methods**. These methods are used for the solution of large linear systems of equations, with typically several millions to several billions of unknowns. The *Service de Métrologie Nucléaire* team has an internationally recognized expertise in the field of multigrid methods. In particular, the team develops the AGMG code, which has hundreds of users around the world.

The basic principle behind multigrid methods is to solve a given linear system using a hierarchy of decreasing in size linear systems. The above subjects within this framework are in general related to *aggregation-based* multigrid methods, which construct a hierarchy of decreasing in size linear systems by grouping unknowns into aggregates ; this yields simple yet efficient multigrid variants. Because the aggregates are automatically formed on the basis of the linear system matrix, these methods are on the other hand much more flexible than the multigrid methods based on a predefined hierarchy of grids.

All master thesis subjects are directly connected to the research activities of the team.

Short Description : next pages

# 1 Algebraic multigrid methods for electromangetism: novel aggregation strategies

The focus of the proposed work is on the solution of linear systems of equations arising from the disretization of boundary value problems of curl-curl type, that is, problems in which the highest order term corresponds to a double application of the **curl** differential operator. The solution of such systems is a key element of numerous simulations in the field of electromagnetism. These systems are characterized by the kernel of the discrete **curl** operator, that corresponds (in the case of simply connected domain) to the space of discrete gradients.

The considered solution method is of aggregation-based algebraic multigrid type. These iterative methods build a hierarchy of grids by grouping unknowns into aggregates, and subsequently use this hierarchy to ensure a rapid convergence. For the curl-curl problems, the aggregation is first performed for the auxiliary unknowns associated with discrete gradients, and the actual aggregates are then deduced from this latter.

The aggregation in the space of discrete gradients is typically based on the projection of the system matrix into this space, projection whose result is, paradoxically, independent on the curl-curl term (since for a regular enough $\mathbf{u}$ one has $\mathbf{curl}\,\mathbf{grad}\,\mathbf{u} = \mathbf{0}$). This approach is therefore problematic if the low-order terms are either missing (since the result of the projection is then nil) or are not representative of the curl-curl term. Based on the recent convergence analysis of the considered method, it is proposed to develop an alternative approach which is based on the non-projected system matrix. The work includes the use of theoretic tools, algorithmic developments, implementation and evaluation of the proposed approaches.

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